Key

Algebra II
Notes 5-4 Obj: Solve radical equations

5-8: Objective: Solving Radical Equations and inequalities

- Isolate the radical expression/rational exponent on one side of the equation.
- Raise both sides of the equation to the same power. (If trying to undo a square root, square both sides. If trying to undo a cube root, cube both sides. If solving for an x raised to the $\frac{2}{3}$ power, raise both sides to the $\frac{3}{2}$, etc.)
- Check for extraneous roots! Plug your answer(s) back into the original equation to make sure they work!

Example 1: Solve

$\sqrt{x+5}-1=3$

$+1 \quad +1$

$(\sqrt{x+5})^2=(4)^2$

$x+5=16$

$x=11$

CK

$\sqrt{11+5}-1=3$

$4-1=3$ ✓

$\sqrt[3]{x}+2=4$

$\sqrt[3]{x}=2$

$(\sqrt[3]{x})^3=2^3$

$x=8$

$\sqrt[3]{8}+2=4$

$2+2=4$ ✓

You try.

$\sqrt{x-2}+3=5$

$\sqrt{x-2}=2$

$(\sqrt{x-2})^2=2^2$

$x-2=4$

$x=6$

$\sqrt{6-2}+3=5$

$2+3=5$ ✓

$\sqrt[3]{x-1}=2$

$(\sqrt[3]{x-1})^3=2^3$

$x-1=8$

$x=9$

$\sqrt[3]{9-1}=2$

$\sqrt[3]{8}=2$ ✓

Example 2. Rewrite the formula. The suspension cables from the Golden Gate Bridge's towers are farther above the roadway near the towers and closer to the roadway near the middle of the bridge. You can figure out your distance from the middle of the bridge, x, in feet, and the height of the suspension cable, y, in feet, at your position by using the equation

$x = \dfrac{\sqrt{y-220}}{0.010583}$. About how far is the cable from the roadway when you are 200 ft from the middle of the bridge?

$(.010583x)^2 = (\sqrt{y-220})^2$

$.000112x^2 \approx y - 220$

$.000112(200)^2 = y - 220$

$y \approx 224.48$ ft.

Example3. Solve.

$(\sqrt{3x-2})^2 = (x-4)^2$ FOIL

$3x - 2 = x^2 - 4x - 4x + 16$

$0 = x^2 - 8x + 16 - 3x + 2$

$0 = x^2 - 11x + 18$

$0 = (x-9)(x-2)$

$x = 9 \quad x = 2$

ch.

$\sqrt{27-2} = 9-4$

$5 = 5$ ✓

$\sqrt{6-2} = 2-4$

$2 \neq -2$ no!

Use your GC to graph and confirm.

You try. $(\sqrt{x+2})^2 = (x+2)^2$

$x + 2 = x^2 + 4x + 4$

$0 = x^2 + 3x + 2$

$(x+2)(x+1)$

$x = -2 \quad x = -1$

ch.

$\sqrt{0} = -2 + 2$ ✓

$\sqrt{1} = -1 + 2$ ✓

$1 = 1$

Example 4. Solve with rational exponents.

$(x^2+5x+5)^{\frac{5}{2}}=1$

use reciprocal power

$\left[(x^2+5x+5)^{5/2}\right]^{2/5}=1^{2/5}$

$x^2+5x+5=1$

$x^2+5x+4=0$

$(x+4)(x+1)=0$

$x=-4 \quad x=-1$

Ch. $(16-20+5)^{5/2}=1$ $(1-5+5)^{5/2}=1$

$1^{5/2}=1$ ✓ $1=1$ ✓

You try. $(x+18)^{\frac{3}{2}}=(x-2)^3$

FIX

$\left((x+18)^{3/2}\right)^{2/3}=\left((x-2)^3\right)^{2/3}$

$x+18=(x-2)^2$ FOIL

$x+18=x^2-4x+4$

$0=x^2-5x-14$

$(x-7)(x+2)=0$

$x=7 \quad x=-2$

ch $(25)^{3/2}=5^3$

5^3 ✓

ch $16^{3/2}=(-4)^3$

$4^3 \neq -4^3$

Example 5. Solve two radicals.

$\sqrt{x+9}-\sqrt{2x}=3$

separate √ then 2 both sides

$\left(\sqrt{x+9}\right)^2=\left(3+\sqrt{2x}\right)^2$ FOIL

$(3+\sqrt{2x})(3+\sqrt{2x})$

$9+3\sqrt{2x}+3\sqrt{2x}+\sqrt{4x^2}$

$9+6\sqrt{2x}+2x$

$x+9=9+6\sqrt{2x}+2x$

$-9 \quad -9$

$x=6\sqrt{2x}+2x$

$-2x \quad -2x$

$(-x)^2=(6\sqrt{2x})^2$

$x^2=36(2x)$

$x^2=72x$

$x^2-72x=0$

$x(x-72)=0$

$x=0 \quad x=72$

ch 0

$\sqrt{9}-\sqrt{0}=3$ ✓

ch 72

$\sqrt{81}-\sqrt{144}=3$

$9-12 \neq 3$

You try. $\sqrt{x+4}-\sqrt{3x}=-2$

$\left(\sqrt{x+4}\right)^2=\left(\sqrt{3x}-2\right)^2$

$x+4=3x-4\sqrt{3x}+4$

$-2x=-4\sqrt{3x}$

$(2x)^2=(4\sqrt{3x})^2$

$4x^2=16(3x)$

$4x^2=48x$

$(\sqrt{3x}-2)(\sqrt{3x}-2)$

$\sqrt{9}x^2-4\sqrt{3x}+4$

$3x-4\sqrt{3x}+4$

$4x^2-48x=0$

$4x(x-12)=0$

$x \neq 0 \quad (x=12)$

ch 0

$\sqrt{4}-\sqrt{0} \neq -2$

$\sqrt{16}-\sqrt{36}=-2$

$4-6=-2$ ✓